A Hybrid Method Solution of Scattering by Conducting Cylinders (TM Case)

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Abstract—**The finite element or finite difference techniques are well known for the solution of Maxwell's equation in differential form. But terminating the mesh accurately at a finite distance from the body in case of an open problem is a major challenge. Previously, the method has been applied for only electrostatic problems. This hybrid method is applied for TM scattering problems and results are documented in this paper. This new approach, as in the electrostatic case, allows for the terminating surface to encapsulate the body very tightly. As before, the finite element technique is used for open region problems whereas the integral equation solution approach using Green's function is applied to enforce the radiation condition. At each iteration cycle, the induced currents on the conducting cylinder are evaluated and their scattered fields at the terminating surface is calculated. Using this method for TM cases, the computational efficiency of the finite element method can be increased. It can be generalized for the case of inhomogeneous and nonlinear media. In this paper numerical results are presented for the solution of Helmholtz's equation to illustrate the accuracy of the technique.**

I. Introduction

The FINITE element method has been used extensively to solve Maxwell's equation in differential form. As it is mentioned in various literature [1], [2], the computational domain for an open region problem has to be reduced for an efficient solution. Therefore, terminating the finite element mesh close to the object boundary is necessary to reduce the number of unknowns (nodes) in the finite element matrix. But one has to be careful about abrupt termination of the mesh, which can introduce errors in the solution. In this paper, the new method keeps the computational domain very small, hence, there are fewer unknowns, and it applies appropriate boundary conditions to the mesh boundary, so that abrupt termination does not have any undesirable effects on the solution.

The MEI technique [3], [6] has been found to give acceptable results, but calculation of the matrix elements could be very demanding. The results presented in [4] are interesting, but convergence is not achieved for certain problems. If the assumed metrons do not represent the charge distribution on the body of interest, the boundary condition is violated. So the solution of partial differential equation may or may not be acceptable because of the approximation introduced by the inexact boundary condition.

This method is applied to electrostatic problems and yielded acceptable results. The purpose of the paper is to demonstrate that the method also works for dynamic problems and to investigate the behavior of it. A simple problem of scattering from a perfectly conducting cylinder, residing in a vacuum, is chosen for that purpose. The cylinder is assumed to be illuminated by a time–harmonic uniform plane wave of angular frequency ω, whose electric field is parallel to the cylinder axis. This incident field produces axially directed induced surface currents on the conductor, an axially directed scattered electric field from that induced current, while the magnetic field is purely transverse to the cylinder axis.

In this paper, the finite element technique in conjunction with the integral equation approach is used to solve Helmholtz's equation for two-dimensional TM scattering problems involving open regions, i.e.,

$$\nabla^2 E_z + k^2 E_z = \frac{\partial^2 E_z(x, y)}{\partial x^2} + \frac{\partial^2 E_z(x, y)}{\partial y^2} + k^2 E_z = 0, \quad x, y \in \mathcal{R} \tag{1}$$

where E_z is the axial component of the electric field, which does not depend on the axial coordinate (z), for the TM case. The free space wavenumber k is given by

$$k = \omega\sqrt{\epsilon_0 \mu_0}.$$

Triangular finite elements are used to approximate an arbitrary boundary accurately. In this approach elements do not extend to infinity, but are limited only to layers from the structure. As it was evident in the electrostatic case, this method encloses the conducting body very tightly without sacrificing the advantages of the finite element methods such as sparsity of the matrix. On one hand, it allows treating problems with infinite domains by using the tools of the integral equation method and on the other hand, it exploits the capabilities of FEM to handle various "irregularities" in the enclosed domain, such as variable coefficients, etc. The artificial outside perimeter of the domain may be chosen to have any shape desired so as to enclose these irregularities efficiently and thus to reduce the size of the computational domain.

Section II describes the theory and the exact procedure of the method. Numerical results of various structures are

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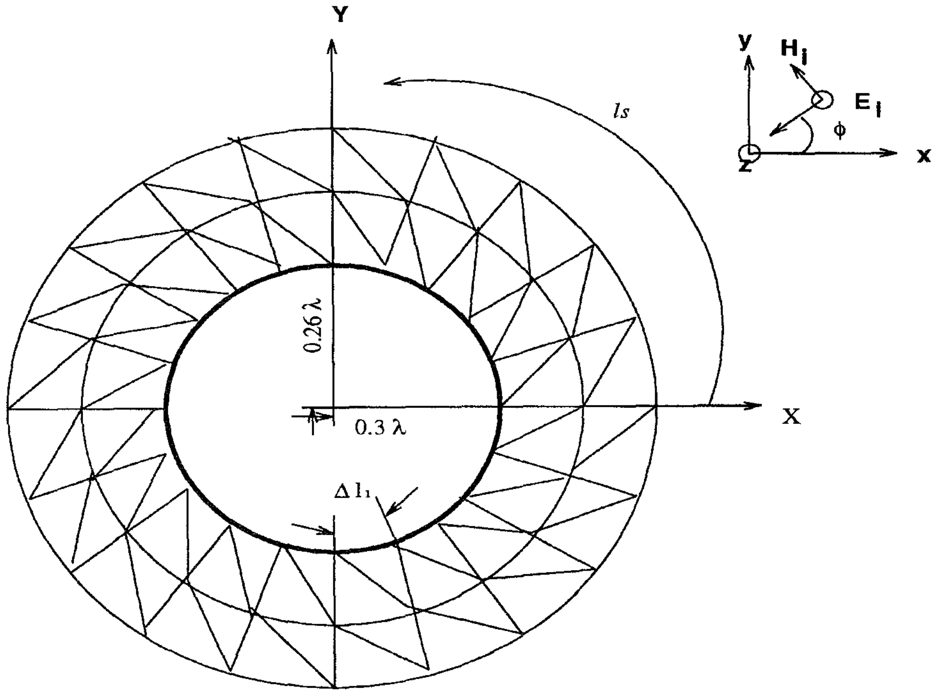

Fig. 1. Finite element mesh for elliptic cylinder.

outlined in Section III. A brief conclusion is given in Section IV.

II. THEORY

In this analysis we assumed a monochromatic plane wave, incident on the conductor, whose electric field is given as (Fig. 1)

$$\overline{E}_\iota = E_0 \exp\{jk(x\cos\phi + y\sin\phi)\}\hat{z} \tag{2}$$

where $\hat{z}$ is the unit vector along z direction. Equation (1) is satisfied by the incident electric field $\overline{E}_\iota$, the scattered electric field ($\overline{E}_s$) produced by the currents induced on the conductor, and the total electric field given as

$$\overline{E} = \overline{E}_\iota + \overline{E}_s. \tag{3}$$

The scattered field $\overline{E}_s$ is given by

$$\overline{E}_s = -j\omega\overline{A} = -j\omega\mu_0 \oint_C \overline{J}_s G(r, r')\, dr' \tag{4}$$

where $\overline{A}$ is the magnetic vector potential, $\overline{J}_s$ is the induced surface current density which is given by

$$\overline{J}_s = J_{sz}\hat{z}$$

dr' is the element of the contour C bounding the conductor cross section, and the Green's function for this problem is given by

$$G(r, r') = \frac{1}{4j} H_0^{(2)}(k|r - r'|) \tag{5}$$

where $H_0^{(2)}$ is the Handel's function of the second kind and order zero. Since J_{sz} is independent of z, there are no charges associated with this current, hence, the electric scalar potential Φ is zero everywhere. The transverse components of the electric and magnetic fields are

$$\overline{E}_t = 0 \tag{6}$$

$$\overline{H}_t = \frac{1}{j\omega\mu_0}\hat{z} \times \overline{\nabla}_t E_z \tag{7}$$

respectively. $\hat{x}$ and $\hat{y}$ are the unit vectors along x and y direction. The transverse nabla operator is given by

$$\overline{\nabla}_t = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}. \tag{8}$$

The transverse components of electric fields are [from (6)]

$$E_x = E_y = 0 \tag{9}$$

and from (7) and (8) we get

$$H_x = -\frac{1}{j\omega\mu_0}\frac{\partial E_z}{\partial y} \tag{10}$$

$$H_y = \frac{1}{j\omega\mu_0}\frac{\partial E_z}{\partial x}. \tag{11}$$

Since the cylinder is perfectly conducting, the boundary condition requires that the tangential components of the total electric field be zero on the conductor. Hence,

$$E_z^{\tan} = 0 \tag{12}$$

on the conductor surface. We can express the surface current density on the conductor in terms of a tangential magnetic field just outside the conductor. Hence, (12) leads us to

$$\overline{J}_s = \hat{n} \times \overline{H}_t^+ \tag{13}$$

where $\hat{n}$ is the unit normal on the conductor surface.

Now the open region surrounding the body is subdivided into nonoverlapping finite elements. Here, the solution region

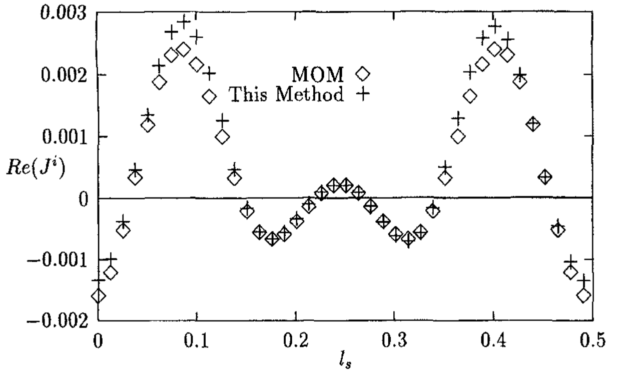

Fig. 2. Real part of the induced current on the elliptic conductor; outer boundary 0.2λ away.

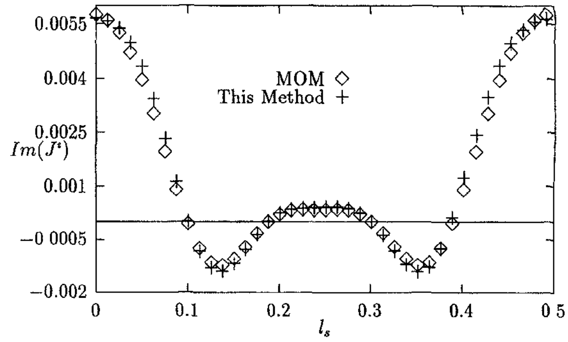

Fig. 3. Imaginary part of the induced current on the elliptic conductor.

does not extend to infinity or to a large distance from the body, but extends only two layers from the structure. The finite elements are chosen to be triangular because structures with an arbitrary boundary can be simulated very efficiently. To find the electric field E_z for the two-dimensional solution region, we seek an approximation for the electric field E_z^e within an element e and then interrelate the fields in various elements such that the electric field is continuous across interelement boundaries [5]. Using a polynomial approximation of an electric field over an element, the solution for the whole region is

$$E_z(x, y) \simeq \sum_{e=1}^{N} E_z^e(x, y) \tag{14}$$

where N is the number of triangular elements into which the solution region is divided. The most common form of approximation for E_z^e within an element is a linear polynomial, and we carried our analysis based on that. Using the same procedure as described in [2], the functional $I(\Phi_e)$, which is the energy per unit length associated with the element e, is given by

$$I(\Phi_e) = \frac{1}{2}\int_s \{|\nabla\Phi_e|^2 - k^2\Phi_e^2\}\, ds \tag{15}$$

where Φ is E_z in our problem. Now we know

$$\nabla\Phi_e(x, y) = \sum_{i=1}^{3} \Phi_e^j \nabla\alpha_e^j \tag{16}$$

where Φ_e^j are the values of the vertices of the element e and α_e^j's which are described in [2]. Now substituting (16) into (15) and the expression for Φ_e over an element we get

$$I(\Phi_e) = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} \Phi_e^i\{C_e^{ij} - k^2 T_e^{ij}\}\Phi_e^j \tag{17}$$

where C_e^{ij} is as defined in [2] and T_e^{ij} is given as

$$T_e^{ij} = \int_s \alpha_i\alpha_j\, ds. \tag{18}$$

Now (17) can be written in matrix form as

$$I(\Phi_e) = \frac{1}{2}[\Phi_e]^t[C_e][\Phi_e] - \frac{k^2}{2}[\Phi_e]^t[T_e][\Phi_e] \tag{19}$$

where t denotes the transpose of the matrix and

$$T_e^{ij} = \begin{cases} \frac{A_e}{6}, & i = j \\ \frac{A_e}{12}, & i \neq j \end{cases} \tag{20}$$

where A_e is the area of the triangular element e. Now assembling all such elements in the solution region, the total energy of the assemblage is given by

$$I(\Phi) = \sum_{e=1}^{N} I(\Phi_e) = \frac{1}{2}[\Phi]^t[C][\Phi] - \frac{k^2}{2}[\Phi]^t[T][\Phi]. \tag{21}$$

The matrices $[C]$ and $[T]$ are the assemblage of individual coefficient matrices $[C_e]$ and $[T_e]$, respectively. The column matrix $[\Phi]$ represents E_z at corresponding nodes.

Equation (19) can be split into two parts for two types of nodes. The first set of nodes are called free nodes on which the electric field needs to be solved for, and the second set are called fixed nodes on which E_zs are known. If all the free nodes are numbered first and the fixed nodes last, we can rewrite (19) as

$$I(\Phi) = \frac{1}{2}[\Phi_f \quad \Phi_p]\left\{\begin{bmatrix} C_{ff} & C_{fp} \\ C_{pf} & C_{pp} \end{bmatrix} - k^2\begin{bmatrix} T_{ff} & T_{fp} \\ T_{pf} & T_{pp} \end{bmatrix}\right\}\begin{bmatrix} \Phi_f \\ \Phi_p \end{bmatrix} \tag{22}$$

where subscripts f and p refer to free and fixed nodes, respectively. Now to find the minimum of the functional $I(\Phi)$ we set

$$\frac{\partial I(\Phi)}{\partial \Phi_f} = 0. \tag{23}$$

In general, it yields

$$\{[C_{ff} \quad C_{fp}] - k^2[T_{ff} \quad T_{fp}]\}\begin{bmatrix} \Phi_f \\ \Phi_p \end{bmatrix} = 0. \tag{24}$$

This equation can be written as

$$\{[C_{ff}] - k^2[T_{ff}]\}[\Phi_f] = \{k^2[T_{fp}] - [C_{fp}]\}[\Phi_p] \tag{25}$$

where $[\Phi_f]$ is our unknown vector.

In this analysis as before [2] the nodes, which are residing on the terminating surface as well as on the conductor boundary, are called fixed nodes. The electric fields on these fixed nodes, residing on the conductor, are known from the boundary condition on the conducting surface, i.e., (12). The electric fields of the nodes, residing on the terminating surface are given by (4) if we know the induced current on the conductor.

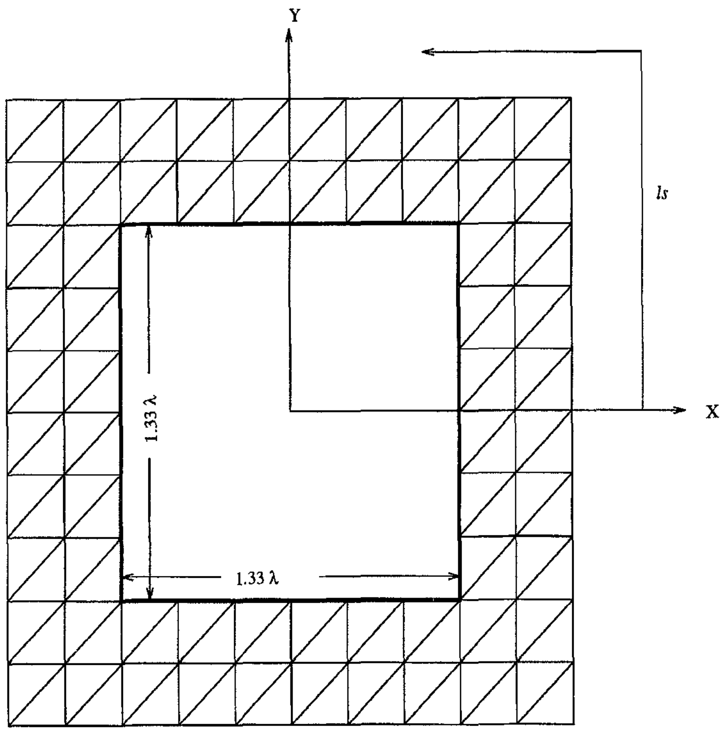

Fig. 4. Finite element mesh for square cylinder.

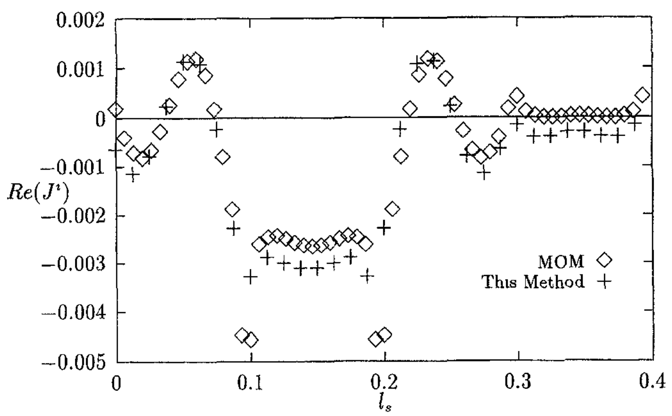

Fig. 5. Real part of the induced current on the square conductor; outer boundary 0.267λ away.

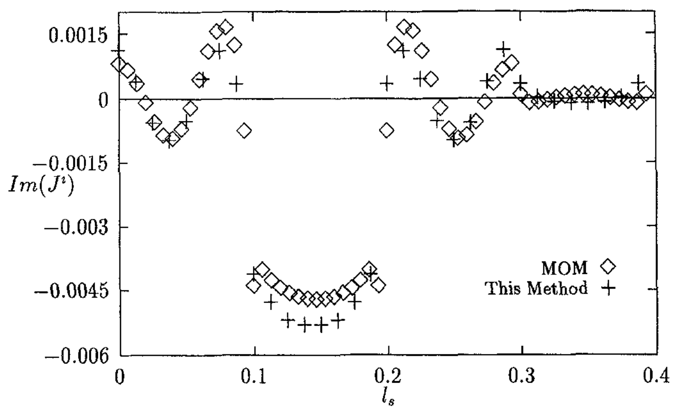

Fig. 6. Imaginary part of the induced current on the square conductor.

The nodes other than fixed nodes are called free nodes for which electric fields will be determined by (25). Suppose we know the field sources within the mesh. These sources are the induced current on the conductor surfaces. These sources produce field in free space, and the scattered field at the terminating surface can be evaluated using (4), given as

$$E_{scat} = -j\omega\mu_0 \sum_i \int_{\Delta l_i} J_{\Delta l_i} \frac{1}{4j} H_0^{(2)}(k|r - r'|)\, dl' \quad (26)$$

here $J_{\Delta l_i}$ is the current distribution over a particular segment on the conductor surface, i.e., Δl_i (Fig. 1), and dl' is the element of that segment.

Equation (1) can be solved for both the scattered field ($\overline{E}_s$) as well as for the total field ($\overline{E}$). If we use the total field criteria, the tangential electric field on the conductor is zero, but in the scattered field formulation it is $-\overline{E}_i$. At the beginning of iteration the electric field at the terminating surface is arbitrarily assumed to be zero for scattered field formulation, but $\overline{E}_i$ for total field formulation. However, for finite size of the meshes, there can be numerical differences between solutions for two cases. In our analysis we used total field formulation. Hence, (12) is valid on the conductor surface, but for the terminating surface the boundary condition will be formulated using total field formulation.

As discussed in [2], at each iteration cycle, the field sources within the mesh domain are found out. These sources are the currents on the conducting surface. Current distribution over a particular segment ($J_{\Delta l_i}$) is calculated through (13) and (10)–(11), which is given as

$$J_z = \frac{1}{j\omega\mu_0} \frac{dE_z}{dn}. \quad (27)$$

To calculate the normal derivative of the electric field we used

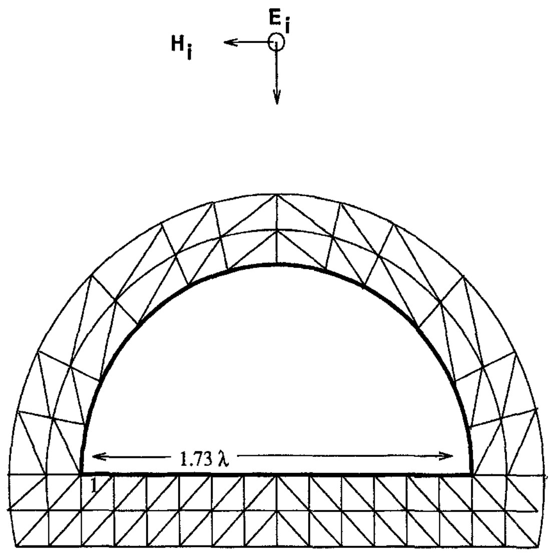

Fig. 7. Finite element mesh for half cylinder.

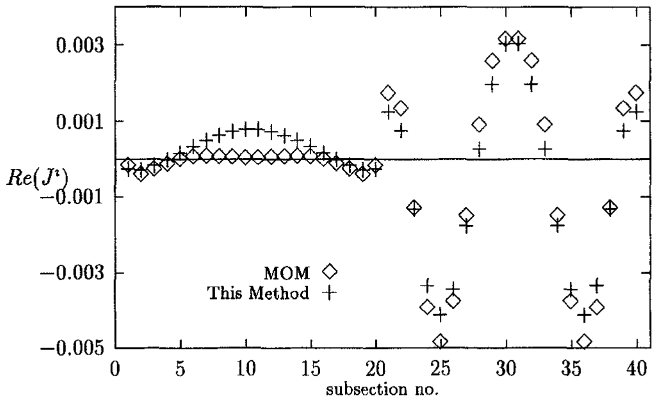

Fig. 8. Real part of the induced current on the half cylinder conductor; outer boundary 0.173λ away.

both first order and second order differences. As expected, the computational accuracy is increased using second order differences. Now replacing (27) in (26) we get

$$E_{scat} = -\frac{1}{4j}\sum_i \int_{\Delta l_i} \left(\frac{dE_z}{dn}\right)_{\Delta l_i} H_0^{(2)}(k|r - r'|)\, dl'. \quad (28)$$

The total field at the mesh termination is now given by

$$E_z^{ob} = E_i^{ob} - \frac{1}{4j}\sum_i \int_{\Delta l_i} \left(\frac{dE_z}{dn}\right)_{\Delta l_i} H_0^{(2)}(k|r - r'|)\, dl' \quad (29)$$

where superscript ob denotes the terminating surface or the outside boundary. Hence, the boundary condition on the terminating surface is "exact."

The iteration continues with a new set of electric fields at the terminating surface replaced by the old values in vector $[\Phi_p]$. If the electric field on the terminating surface at kth iteration is denoted by $[E_{ob}^k]$, then the electric field at the terminating surface at $(k+1)$th iteration is given by

$$[E_{ob}^{k+1}] \leftarrow [E_{ob}^k] + \alpha\{[E_{ob}^{k+1}] - [E_{ob}^k]\}. \quad (30)$$

An under-relaxation factor, α, of 0.1–1.0 is chosen to get convergence for the nodes residing on the outside boundary

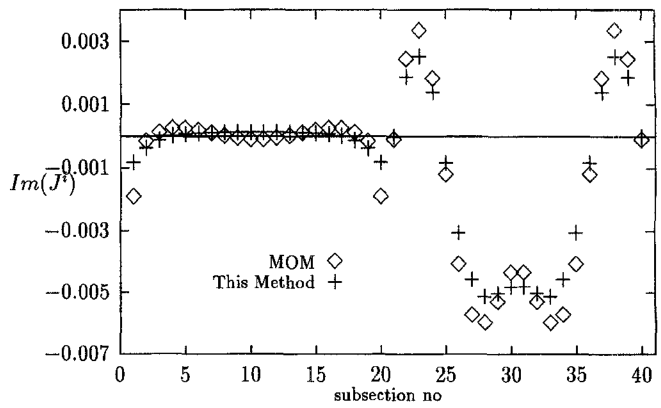

Fig. 9. Imaginary part of the induced current on the half cylinder conductor.

or the terminating surface. The iterative procedure is stopped when the electric field at the terminating surface attains some specified error criteria, hence, the boundary condition at the terminating surface is "numerically exact."

The electric field for free nodes are updated at each iteration as given by

$$[E_f^{k+1}] \leftarrow [E_f^k] + \beta\{[E_f^{k+1}] - [E_f^k]\} \quad (31)$$

where $[E_f^k]$ denotes electric field of free nodes at kth iteration. For free nodes an over-relaxation factor, β, of 1–1.9 is chosen for convergence. This procedure is simple to use, and convergence is achieved for all the structures that this new method was applied to.

The proposed method can be regarded as a hybrid of differential and integral equation techniques because a finite element approach is applied for the electric field within the domain, and a boundary condition expressed in terms of an integral (containing the unknown source distribution under the integral) is formulated for the terminating surface. If the terminating surface collapses on the conductor surface, a boundary element integral equation approach is obtained because the quantities to be solved for are only the surface currents, and they are solved from an integral equation.

III. Numerical Results

Let us consider the case of an elliptic cylinder of major axis 0.6λ and minor axis 0.52λ. A two-layer finite element mesh for the problem is shown in Fig. 1. There are 80 fixed nodes—40 of them are on the inside boundary and the other 40 nodes are on the outside boundary or the terminating surface. Another 40 free nodes are elements of our unknown vector. In this case we have used total field formulation. The iteration starts by assuming $\overline{E}_i$ on the outside boundary. The incident field is a uniform plane wave with $\phi = 0$ and unit amplitude ($E_0 = 0$). Current distribution on a particular segment of the body is calculated using second-order difference. The real and imaginary part of the induced current on the conductor is plotted on Figs. 2 and 3, respectively, along with method of moments (MOM) [7] values. For MOM, 40 subsections are chosen on the conducting surfaces. The currents are found out for the MOM using pulse basis and point matching testing procedures. As it is evident, it agrees very well with MOM values of the induced current.

As a second example consider the case of a square cylinder of side 1.33λ. The finite element grid of this problem is shown in Fig. 4. In this case there are 40 unknowns in the form of free nodes and 88 fixed nodes on which the fields are known at each iteration cycle. Forty fixed nodes form the contour of the conductor on which the tangential electric field is zero for total field formulation. The iteration starts assuming $\overline{E}_i$ on the outside boundary which is comprised of 48 fixed nodes. Current distribution on a particular segment of the body is calculated using second order difference. The real and imaginary parts of the induced current on the conductor are plotted on Figs. 5 and 6, respectively, along with MOM values. Forty subsections are chosen to calculate induced current using MOM. As before, pulse basis functions and the point matching testing procedure are used to evaluate the current on the conductor. The results agree well with MOM values of the induced current, but the current values at corners are widely different from the MOM values. This is expected because of the very inherent nature of FEM where using nodes as unknowns results in convergence problems. As the field blows up at the corner, modeling them with a point value is extremely difficult.

As a last example, consider the case of a half cylinder of diameter 1.73λ. The finite element meshing of this structure is shown in Fig. 7. The incident field in this case comes from the convex side. In this problem electric field values are found out for 44 free nodes at each iteration cycle. The tangential electric field on the scatterer, formed by 40 fixed nodes, is zero for total field formulation. The terminating surface also in this case is two layers away from the conductor. As before, the iteration starts by assuming $\overline{E}_i$ on the outside boundary, which is comprised of 48 fixed nodes. To calculate the normal derivative of the electric field, the second-order difference is used, which gives the current distribution over a particular segment. The real and imaginary parts of the induced current on the conductor are plotted on Figs. 8 and 9, respectively, along with MOM values. Currents are plotted from node 1 in an anti-clockwise sense. Forty subsections are chosen to calculate the induced current using MOM. As before, pulse basis functions and the point matching testing procedure are used to evaluate the current on the conductor. The results agree well with MOM values of the induced current, but the real part of the current values at the shadow region of the conductor is in variance with MOM values. Since the current value is very low in this region, numerical error is introduced while differencing two small numbers.

IV. Conclusion

A hybrid method is presented for the solution of Helmholtz's equation in two dimensions for open region TM scattering problems. The results are in reasonable agreement with the solution of MOM. This method yields a highly sparse matrix and can be used very effectively in solving electromagnetic scattering problems for inhomogeneous and nonlinear media.

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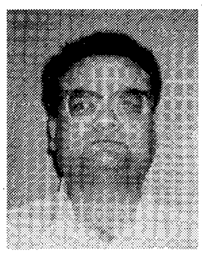

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